Section and Mid-Point Formula

Exercise 13A

Question 1.
Calculate the co-ordinates of the point P which divides the line segment joining:
(i) A (1, 3) and B (5, 9) in the ratio 1: 2.
(ii) A (-4, 6) and B (3, -5) in the ratio 3: 2.

Solution:

(i) Let the co-ordinates of the point P be (x, y).

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 5 + 2 \times 1}{1 + 2} = \frac{7}{3}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 9 + 2 \times 3}{1 + 2} = \frac{15}{3} = 5$$

Thus, the co-ordinates of point P are $\left(\frac{7}{3}, 5\right)$.

(ii) Let the co-ordinates of the point P be (x, y).

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{3 \times 3 + 2 \times (-4)}{3 + 2} = \frac{1}{5}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{3 \times (-5) + 2 \times 6}{3 + 2} = \frac{-3}{5}$$

Thus, the co-ordinates of point P are $\left(\frac{1}{5}, -\frac{3}{5}\right)$.

Question 2.
In what ratio is the line joining (2, -3) and (5, 6) divided by the x-axis.

Solution:
Let the line joining points A (2, -3) and B (5, 6) be divided by point P (x, 0) in the ratio k: 1.

$$y = \frac{ky_2 + y_1}{k + 1}$$

$$0 = \frac{k \times 6 + 1 \times (-3)}{k + 1}$$

$$0 = 6k - 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Question 3.
In what ratio is the line joining (2, -4) and (-3, 6) divided by the y-axis.

Solution:

Let the line joining points A (2, -4) and B (-3, 6) be divided by point P (0, y) in the ratio k: 1.

$$x = \frac{kx_2 + x_1}{k + 1}$$

$$0 = \frac{k \times (-3) + 1 \times 2}{k + 1}$$

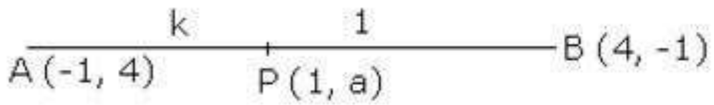
$$0 = -3k + 2$$

$$k = \frac{2}{3}$$

Thus, the required ratio is 2: 3.

Question 4.
In what ratio does the point (1, a) divided the join of (-1, 4) and (4, -1)? Also, find the value of a.

Solution:



Let the point P (1, a) divides the line segment AB in the ratio k: 1.
Using section formula, we have:

$$1 = \frac{4k - 1}{k + 1},$$

$$\Rightarrow k + 1 = 4k - 1$$

$$\Rightarrow 2 = 3k$$

$$\Rightarrow k = \frac{2}{3} \quad \dots(1)$$

$$\Rightarrow a = \frac{-k + 4}{k + 1}$$

$$\Rightarrow a=\frac{\frac{-2}{3}+4}{\frac{2}{3}+1} \quad (\text{from}(1))$$

$$\Rightarrow a=\frac{10}{5}=2$$

Hence, the required ratio is 2 : 3 and the value of a is 2.

Question 5.
In what ratio does the point (a, 6) divide the join of (-4, 3) and (2, 8)? Also, find the value of a.

Solution:
Let the point P (a, 6) divides the line segment joining A (-4, 3) and B (2, 8) in the ratio k: 1.

Using section formula, we have:

$$6=\frac{8k+3}{k+1},$$

$$\Rightarrow 6k+6=8k+3$$

$$\Rightarrow 3=2k$$

$$\Rightarrow k=\frac{3}{2} \quad \ldots(1)$$

$$\Rightarrow a=\frac{2k-4}{k+1}$$

$$\Rightarrow a=\frac{2\times\frac{3}{2}-4}{\frac{3}{2}+1} \quad (\text{from}(1))$$

$$\Rightarrow a=-\frac{2}{5}$$

Hence, the required ratio is 3 : 2 and the value of a is $-\frac{2}{5}$.

Question 6.
In what ratio is the join of (4, 3) and (2, -6) divided by the x-axis. Also, find the co-ordinates of the point of intersection.

Solution:

Let the point P (x, 0) on x-axis divides the line segment joining A (4, 3) and B (2, -6) in the ratio k: 1.

Using section formula, we have:

$$0 = \frac{-6k + 3}{k + 1}$$

$$0 = -6k + 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Also, we have:

$$x = \frac{2k + 4}{k + 1}$$

$$= \frac{2 \times \frac{1}{2} + 4}{\frac{1}{2} + 1}$$

$$= \frac{10}{3}$$

Thus, the required co-ordinates of the point of intersection are $\left(\frac{10}{3}, 0\right)$.

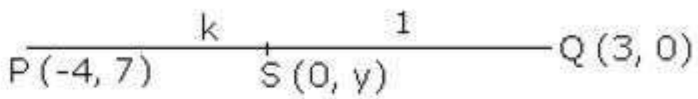
Question 7.

Find the ratio in which the join of (-4, 7) and (3, 0) is divided by the y-axis. Also, find the coordinates of the point of intersection.

Solution:

Let S (0, y) be the point on y-axis which divides the line segment PQ in the ratio k: 1.

Using section formula, we have:



$$0 = \frac{3k - 4}{k + 1}$$

$$\Rightarrow 3k = 4$$

$$k = \frac{4}{3} \quad \ldots(1)$$

$$y = \frac{0+7}{k+1}$$

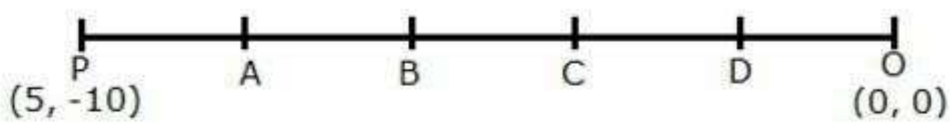
$$y = \frac{7}{\frac{4}{3}+1} \quad (\text{from}(1))$$

$$y = 3$$

Hence, the required ratio is 4 : 3 and the required point is S(0, 3).

Question 8.
Points A, B, C and D divide the line segment joining the point (5, -10) and the origin in five equal parts. Find the co-ordinates of A, B, C and D.

Solution:



Point A divides PO in the ratio 1: 4.
Co-ordinates of point A are:

$$\left(\frac{1\times0+4\times5}{1+4}, \frac{1\times0+4\times(-10)}{1+4}\right) = \left(\frac{20}{5}, \frac{-40}{5}\right) = (4,-8)$$

Point B divides PO in the ratio 2: 3.
Co-ordinates of point B are:

$$\left(\frac{2\times0+3\times5}{2+3}, \frac{2\times0+3\times(-10)}{2+3}\right) = \left(\frac{15}{5}, \frac{-30}{5}\right) = (3,-6)$$

Point C divides PO in the ratio 3: 2.
Co-ordinates of point C are:

$$\left(\frac{3\times0+2\times5}{3+2}, \frac{3\times0+2\times(-10)}{3+2}\right) = \left(\frac{10}{5}, \frac{-20}{5}\right) = (2,-4)$$

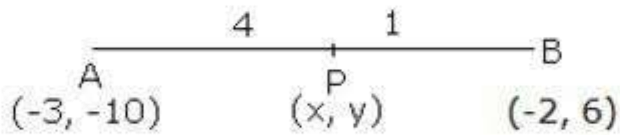
Point D divides PO in the ratio 4: 1.
Co-ordinates of point D are:

$$\left(\frac{4\times0+1\times5}{4+1}, \frac{4\times0+1\times(-10)}{4+1}\right) = \left(\frac{5}{5}, \frac{-10}{5}\right) = (1,-2)$$

Question 9.
The line joining the points A (-3, -10) and B (-2, 6) is divided by the point P such that $\frac{PB}{AB} = \frac{1}{5}$ Find the co-ordinates of P.

Solution:

Let the co-ordinates of point P are (x, y).



Given : PB : AB = 1 : 5

$\therefore$ PB : PA = 1 : 4

Coordinates of P are

$$(x, y) = \left(\frac{4\times(-2)+1\times(-3)}{5}, \frac{4\times6+1\times(-10)}{5}\right) = \left(-\frac{11}{5}, \frac{14}{5}\right)$$

Question 10.
P is a point on the line joining A (4, 3) and B (-2, 6) such that 5AP = 2BP. Find the co-ordinates of P.

Solution:

5AP = 2BP

$$\frac{AP}{BP} = \frac{2}{5}$$

The co-ordinates of the point P are

$$\left(\frac{2\times(-2)+5\times4}{2+5}, \frac{2\times6+5\times3}{2+5}\right)$$

$$\left(\frac{16}{7}, \frac{27}{7}\right)$$

Question 11.
Calculate the ratio in which the line joining the points (-3, -1) and (5, 7) is divided by the line x = 2. Also, find the co-ordinates of the point of intersection.

Solution:
The co-ordinates of every point on the line x = 2 will be of the type (2, y).
Using section formula, we have:

$$x = \frac{m_1 \times 5 + m_2 \times (-3)}{m_1 + m_2}$$

$$2 = \frac{5m_1 - 3m_2}{m_1 + m_2}$$

$$2m_1 + 2m_2 = 5m_1 - 3m_2$$

$$5m_2 = 3m_1$$

$$\frac{m_1}{m_2} = \frac{5}{3}$$

Thus, the required ratio is 5: 3.

$$y = \frac{m_1 \times 7 + m_2 \times (-1)}{m_1 + m_2}$$

$$y = \frac{5 \times 7 + 3 \times (-1)}{5 + 3}$$

$$y = \frac{35 - 3}{8}$$

$$y = \frac{32}{8} = 4$$

Thus, the required co-ordinates of the point of intersection are (2, 4).

Question 12.
Calculate the ratio in which the line joining A (6, 5) and B (4, -3) is divided by the line y = 2.

Solution:

The co-ordinates of every point on the line y = 2 will be of the type (x, 2).
Using section formula, we have:

$$y = \frac{m_1 \times (-3) + m_2 \times 5}{m_1 + m_2}$$

$$2 = \frac{-3m_1 + 5m_2}{m_1 + m_2}$$

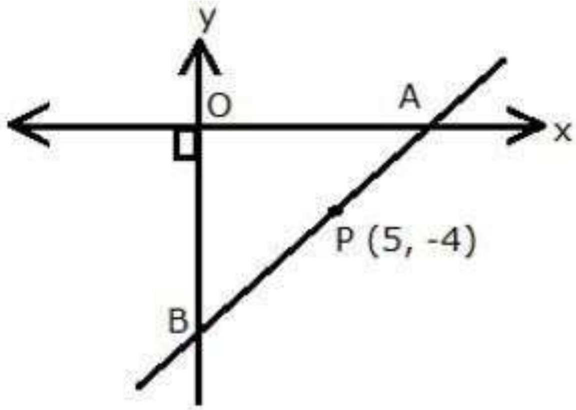
$$2m_1 + 2m_2 = -3m_1 + 5m_2$$

$$5m_1 = 3m_2$$

$$\frac{m_1}{m_2} = \frac{3}{5}$$

Thus, the required ratio is 3: 5.

Question 13.
The point P (5, -4) divides the line segment AB, as shown in the figure, in the ratio 2: 5. Find the co-ordinates of points A and B.



Solution:

Point A lies on x-axis. So, let the co-ordinates of A be (x, 0).
Point B lies on y-axis. So, let the co-ordinates of B be (0, y).
P divides AB in the ratio 2: 5.
We have:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$5 = \frac{2 \times 0 + 5 \times x}{2 + 5}$$

$$5 = \frac{5x}{7}$$

$$x = 7$$

Thus, the co-ordinates of point A are (7, 0).

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$-4 = \frac{2 \times y + 5 \times 0}{2 + 5}$$

$$-4 = \frac{2y}{7}$$

$$-2 = \frac{y}{7}$$

$$y = -14$$

Thus, the co-ordinates of point B are (0, -14).

Question 14.
Find the co-ordinates of the points of trisection of the line joining the points (-3, 0) and (6, 6).

Solution:
Let P and Q be the point of trisection of the line segment joining the points A (-3, 0) and B (6, 6).

So, AP = PQ = QB

We have AP: PB = 1: 2
Co-ordinates of the point P are

$$\left(\frac{1\times 6+2\times(-3)}{1+2}, \frac{1\times 6+2\times 0}{1+2}\right)$$

$$=\left(\frac{6-6}{3}, \frac{6}{3}\right)$$

$$=(0,2)$$

We have AQ: QB = 2: 1
Co-ordinates of the point Q are

$$\left(\frac{2\times 6+1\times(-3)}{2+1}, \frac{2\times 6+1\times 0}{2+1}\right)$$

$$=\left(\frac{9}{3}, \frac{12}{3}\right)$$

$$=(3,4)$$

Question 15.
Show that the line segment joining the points (-5, 8) and (10, -4) is trisected by the co-ordinate axes.

Solution:
Let P and Q be the point of trisection of the line segment joining the points A (-5, 8) and B (10, -4).

So, AP = PQ = QB

We have AP: PB = 1: 2

Co-ordinates of the point P are

$$\left(\frac{1\times 10+2\times(-5)}{1+2}, \frac{1\times(-4)+2\times 8}{1+2}\right)$$

$$=\left(\frac{10-10}{3}, \frac{12}{3}\right)$$

$$=(0,4)$$

So, point P lies on the y-axis.

We have AQ: QB = 2: 1

Co-ordinates of the point Q are

$$\left(\frac{2\times 10+1\times(-5)}{2+1}, \frac{2\times(-4)+1\times 8}{2+1}\right)$$

$$=\left(\frac{20-5}{3}, \frac{-8+8}{3}\right)$$

$$=(5,0)$$

So, point Q lies on the x-axis.

Hence, the line segment joining the given points A and B is trisected by the co-ordinate axes.

Question 16.
Show that A (3, -2) is a point of trisection of the line-segment joining the points (2, 1) and (5, -8). Also, find the co-ordinates of the other point of trisection.

Solution:
Let A and B be the point of trisection of the line segment joining the points P (2, 1) and Q (5, -8).

So, PA = AB = BQ

We have PA: AQ = 1: 2

Co-ordinates of the point A are

$$\left(\frac{1\times5+2\times2}{1+2}, \frac{1\times(-8)+2\times1}{1+2}\right)$$

$$=\left(\frac{9}{3}, \frac{-6}{3}\right)$$

$$=(3,-2)$$

Hence, A (3, -2) is a point of trisection of PQ.

We have PB: BQ = 2: 1

Co-ordinates of the point B are

$$\left(\frac{2\times5+1\times2}{2+1}, \frac{2\times(-8)+1\times1}{2+1}\right)$$

$$=\left(\frac{10+2}{3}, \frac{-16+1}{3}\right)$$

$$=(4,-5)$$

Question 17.

If A = (-4, 3) and B = (8, -6)

(i) Find the length of AB.

(ii) In what ratio is the line joining A and B, divided by the x-axis?

Solution:

(i) A (-4,3) and B (8, -6)

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(8+4)^2+(-6-3)^2}$$

$$= \sqrt{144+81}$$

$$= \sqrt{225}$$

$$= 15 \text{ units}$$

(ii) Let P be the point, which divides AB on the x-axis in the ratio k : 1.

Therefore, y-co-ordinate of P = 0.

$$\Rightarrow \frac{-6k+3}{k+1} = 0$$

$$\Rightarrow -6k+3=0$$

$$\Rightarrow k = \frac{1}{2}$$

∴ Required ratio is 1: 2.

Question 18.
The line segment joining the points M (5, 7) and N (-3, 2) is intersected by the y-axis at point L. Write down the abscissa of L. Hence, find the ratio in which L divides MN. Also, find the co-ordinates of L.

Solution:

Since, point L lies on y-axis, its abscissa is 0.
Let the co-ordinates of point L be (0, y). Let L divides MN in the ratio k: 1.

Using section formula, we have:

$$x = \frac{k \times (-3) + 1 \times 5}{k + 1}$$

$$0 = \frac{-3k + 5}{k + 1}$$

$$-3k + 5 = 0$$

$$k = \frac{5}{3}$$

Thus, the required ratio is 5: 3.

$$\text{Now, } y = \frac{k \times 2 + 1 \times 7}{k + 1}$$

$$= \frac{\frac{5}{3} \times 2 + 7}{\frac{5}{3} + 1}$$

$$= \frac{10 + 21}{5 + 3}$$

$$= \frac{31}{8}$$

Question 19.
A (2, 5), B (-1, 2) and C (5, 8) are the co-ordinates of the vertices of the triangle ABC. Points P and Q lie on AB and AC respectively, such that AP: PB = AQ: QC = 1: 2.

(i) Calculate the co-ordinates of P and Q.
(ii) Show that PQ = 1/3 BC.

Solution:

(i) Co-ordinates of P are

$$\left(\frac{1\times(-1)+2\times2}{1+2}, \frac{1\times2+2\times5}{1+2}\right)$$

$$=\left(\frac{3}{3}, \frac{12}{3}\right)$$

$$=(1, 4)$$

Co-ordinates of Q are

$$\left(\frac{1\times5+2\times2}{1+2}, \frac{1\times8+2\times5}{1+2}\right)$$

$$=\left(\frac{9}{3}, \frac{18}{3}\right)$$

$$=(3,6)$$

(ii) Using distance formula, we have:

$$BC = \sqrt{(5+1)^2+(8-2)^2} = \sqrt{36+36} = 6\sqrt{2}$$

$$PQ = \sqrt{(3-1)^2+(6-4)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Hence, $PQ = \frac{1}{3}BC$.

Question 20.

A (-3, 4), B (3, -1) and C (-2, 4) are the vertices of a triangle ABC. Find the length of line segment AP, where point P lies inside BC, such that BP: PC = 2: 3.

Solution:

BP: PC = 2: 3

Co-ordinates of P are

$$\left(\frac{2\times(-2)+3\times3}{2+3}, \frac{2\times4+3\times(-1)}{2+3}\right)$$

$$=\left(\frac{-4+9}{5}, \frac{8-3}{5}\right)$$

$$=(1,1)$$

Using distance formula, we have:

$$AP = \sqrt{(1+3)^2+(1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

Question 21.
The line segment joining A (2, 3) and B (6, -5) is intercepted by x-axis at the point K. Write down the ordinate of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K.

Solution:

Since, point K lies on x-axis, its ordinate is 0.
Let the point K (x, 0) divides AB in the ratio k: 1.
We have,

$$y = \frac{k \times (-5) + 1 \times 3}{k+1}$$

$$0 = \frac{-5k + 3}{k+1}$$

$$k = \frac{3}{5}$$

Thus, K divides AB in the ratio 3: 5.
Also, we have:

$$x = \frac{k \times 6 + 1 \times 2}{k+1}$$

$$x = \frac{\frac{3}{5} \times 6 + 2}{\frac{3}{5} + 1}$$

$$x = \frac{18 + 10}{3+5}$$

$$x = \frac{28}{8} = \frac{7}{2} = 3\frac{1}{2}$$

Thus, the co-ordinates of the point K are $\left(3\frac{1}{2}, 0\right)$.

Question 22.
The line segment joining A (4, 7) and B (-6, -2) is intercepted by the y-axis at the point K. Write down the abscissa of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K.

Solution:

Since, point K lies on y-axis, its abscissa is 0.
Let the point K (0, y) divides AB in the ratio k: 1.
We have,

$$x = \frac{k \times (-6) + 1 \times 4}{k + 1}$$

$$0 = \frac{-6k + 4}{k + 1}$$

$$k = \frac{4}{6} = \frac{2}{3}$$

Thus, K divides AB in the ratio 2: 3.
Also, we have:

$$y = \frac{k \times (-2) + 1 \times 7}{k + 1}$$

$$y = \frac{-2k + 7}{k + 1}$$

$$y = \frac{-2 \times \frac{2}{3} + 7}{\frac{2}{3} + 1}$$

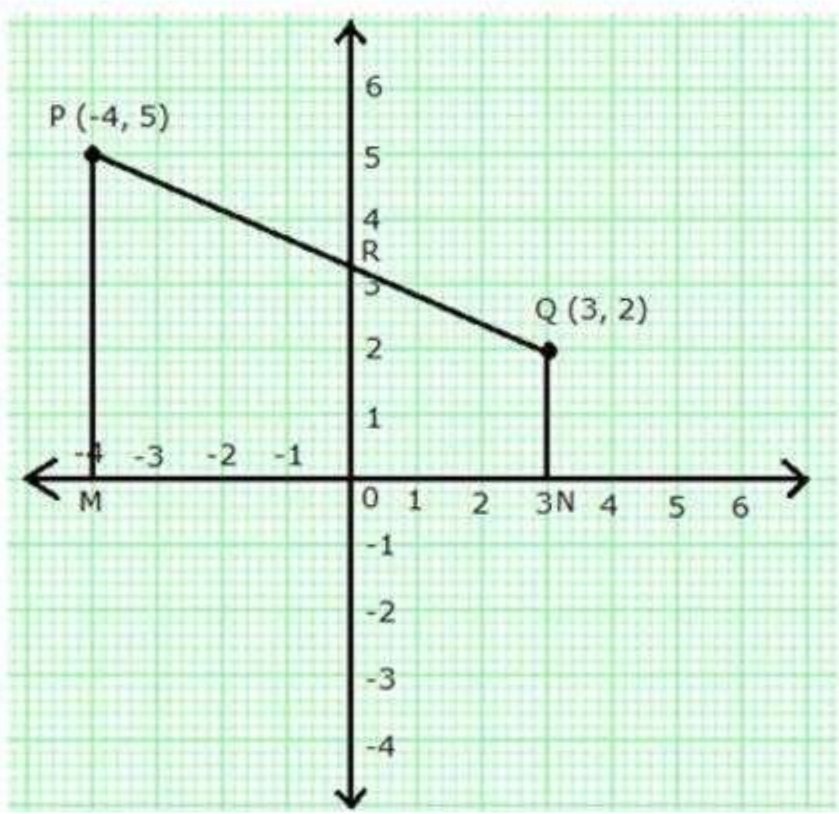
$$y = \frac{-4 + 21}{2 + 3}$$

$$y = \frac{17}{5}$$

Thus, the co-ordinates of the point K are $\left(0, \frac{17}{5}\right)$.

Question 23.
The line joining P (-4, 5) and Q (3, 2) intersects the y-axis at point R. PM and QN are perpendiculars from P and Q on the x-axis. Find:
(i) the ratio PR: RQ.
(ii) the co-ordinates of R.
(iii) the area of the quadrilateral PMNQ.

Solution:



(i) Let point R (0, y) divides PQ in the ratio k: 1.
We have:

$$x = \frac{k \times 3 + 1 \times (-4)}{k + 1}$$

$$0 = \frac{3k - 4}{k + 1}$$

$$0 = 3k - 4$$

$$k = \frac{4}{3}$$

Thus, PR: RQ = 4: 3
(ii) Also, we have:

$$y = \frac{k \times 2 + 1 \times 5}{k + 1}$$

$$y = \frac{2k + 5}{k + 1}$$

$$y = \frac{2 \times \frac{4}{3} + 5}{\frac{4}{3} + 1}$$

$$y = \frac{8+15}{4+3}$$

$$y = \frac{23}{7}$$

Thus, the co-ordinates of point R are $\left(0, \frac{23}{7}\right)$.

(iii) Area of quadrilateral PMNQ

$= \frac{1}{2} \times (PM + QN) \times MN$

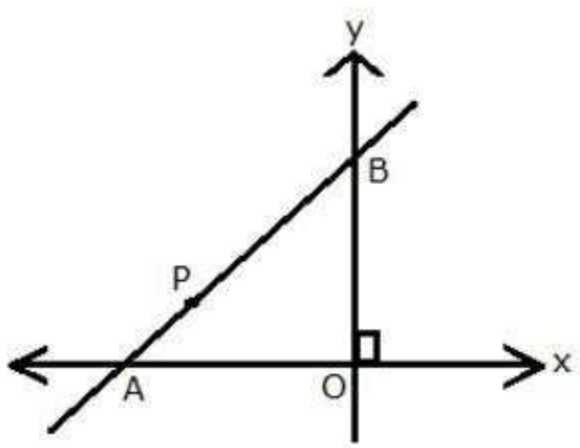
$= \frac{1}{2} \times (5 + 2) \times 7$

$= \frac{1}{2} \times 7 \times 7$

$= 24.5$ sq units

Question 24.

In the given figure, line APB meets the x-axis at point A and y-axis at point B. P is the point (-4, 2) and AP: PB = 1: 2. Find the co-ordinates of A and B.



Solution:

Given, A lies on x-axis and B lies on y-axis.
Let the co-ordinates of A and B be (x, 0) and (0, y) respectively.

Given, P is the point (-4, 2) and AP: PB = 1: 2.

Using section formula, we have:

$$-4 = \frac{1 \times 0 + 2 \times x}{1+2}$$

$$-4 = \frac{2x}{3}$$

$$x = \frac{-4 \times 3}{2} = -6$$

Also,

$$2 = \frac{1 \times y + 2 \times 0}{1+2}$$

$$2 = \frac{y}{3}$$

$$y = 6$$

Thus, the co-ordinates of points A and B are (-6, 0) and (0, 6) respectively.

Question 25.

Given a line segment AB joining the points A(-4, 6) and B(8, -3). Find:

(i) the ratio in which AB is divided by the y-axis

(ii) find the coordinates of the point of intersection

(iii) the length of AB

Solution:

(i)

Let the required ratio be $m_1 : m_2$

Consider $A(-4, 6) = (x_1, y_1)$; $B(8, -3) = (x_2, y_2)$ and let

$P(x, y)$ be the point of intersection of the line sement

and the y-axis.

By section formula, we have,

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2},\quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2},\quad y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

The equation of the y-axis is $x = 0$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2} = 0$$

$$\Rightarrow 8m_1 - 4m_2 = 0$$

$\Rightarrow 8m_1 = 4m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{4}{8}$

$\Rightarrow \frac{m_1}{m_2} = \frac{1}{2}$

(ii)

From the previous subpart, we have,

$\frac{m_1}{m_2} = \frac{1}{2}$

$\Rightarrow m_1 = k$ and $m_2 = 2k$, where k is any constant.

Also, we have,

$x = \frac{8m_1 - 4m_2}{m_1 + m_2}$, $y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$

$\Rightarrow x = \frac{8 \times k - 4 \times 2k}{k + 2k}$, $y = \frac{-3 \times k + 6 \times 2k}{k + 2k}$

$\Rightarrow x = \frac{8k - 8k}{3k}$, $y = \frac{-3k + 12k}{3k}$

$\Rightarrow x = \frac{0}{3k}$, $y = \frac{9k}{3k}$

$\Rightarrow x = 0$, $y = 3$

Thus, the point of intersection is P (0, 3)

(iii)

The length of AB = Distance between two points A and B.

The distance between two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by,

$$\text{Distance AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8 + 4)^2 + (-3 - 6)^2}$$

$$= \sqrt{(12)^2 + (9)^2}$$

$$= \sqrt{144 + 81}$$

$$= \sqrt{225}$$

$$= 15 \text{ units}$$

Question 26.
If P(-b, 9a – 2) divides the line segment joining the points A(-3, 3a + 1) and B(5, 8a) in the ratio 3: 1, find the values of a and b.

Solution:

Take (x_1, y_1) = (-3, 3a + 1) ; (x_2, y_2) = B(5, 8a) and
(x, y) = (-b, 9a – 2)

Here $m_1 = 3$ and $m_2 = 1$

$$\therefore \text{ Coordinate of } P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\Rightarrow x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow -b = \frac{3 \times 5 + 1 \times (-3)}{3 + 1} \text{ and } 9a - 2 = \frac{3 \times 8a + 1(3a + 1)}{3 + 1}$$

$$\Rightarrow -b = \frac{15 - 3}{4} \text{ and } 9a - 2 = \frac{24a + 3a + 1}{4}$$

$$\Rightarrow -4b = 12 \text{ and } 36a - 8 = 27a + 1$$

$$\Rightarrow b = -3 \text{ and } 9a = 9$$

$$\therefore a = 1 \text{ and } b = -3$$

Exercise 13B

Question 1.
Find the mid-point of the line segment joining the points:
(i) (-6, 7) and (3, 5)
(ii) (5, -3) and (-1, 7)

Solution:

(i) A (-6, 7) and B (3, 5)

$$\text{Mid-point of AB} = \left(\frac{-6+3}{2}, \frac{7+5}{2}\right) = \left(\frac{-3}{2}, 6\right)$$

(ii) A (5, -3) and B (-1, 7)

$$\text{Mid-point of AB} = \left(\frac{5-1}{2}, \frac{-3+7}{2}\right) = (2, 2)$$

Question 2.
Points A and B have co-ordinates (3, 5) and (x, y) respectively. The mid-point of AB is (2, 3). Find the values of x and y.

Solution:

Mid-point of AB = (2, 3)

$$\therefore \left(\frac{3+x}{2}, \frac{5+y}{2}\right) = (2, 3)$$

$$\Rightarrow \frac{3+x}{2} = 2 \quad \text{and} \quad \frac{5+y}{2} = 3$$

$$\Rightarrow 3 + x = 4 \quad \text{and} \quad 5 + y = 6$$

$$\Rightarrow x = 1 \quad \text{and} \quad y = 1$$

Question 3.
A (5, 3), B (-1, 1) and C (7, -3) are the vertices of triangle ABC. If L is the mid-point of AB and M is the mid-point of AC, show that LM = 1/2 BC.

Solution:

Given, L is the mid-point of AB and M is the mid-point of AC.
Co-ordinates of L are

$$\left(\frac{5-1}{2}, \frac{3+1}{2}\right) = (2, 2)$$

Co-ordinates of M are

$$\left(\frac{5+7}{2}, \frac{3-3}{2}\right) = (6, 0)$$

Using distance formula, we have:

$$BC = \sqrt{(7+1)^2 + (-3-1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$LM = \sqrt{(6-2)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Hence, } LM = \frac{1}{2}BC$$

Question 4.
Given M is the mid-point of AB, find the co-ordinates of:
(i) A; if M = (1, 7) and B = (-5, 10)
(ii) B; if A = (3, -1) and M = (-1, 3).

Solution:

(i) Let the co-ordinates of A be (x, y).

$$\therefore (1,7) = \left(\frac{x-5}{2}, \frac{y+10}{2}\right)$$

$$\Rightarrow 1 = \frac{x-5}{2} \quad \text{and} \quad 7 = \frac{y+10}{2}$$

$$\Rightarrow 2 = x-5 \quad \text{and} \quad 14 = y+10$$

$$\Rightarrow x = 7 \quad \text{and} \quad y = 4$$

Hence, the co-ordinates of A are (7, 4).

(ii) Let the co-ordinates of B be (x, y).

$$\therefore (-1,3) = \left(\frac{3+x}{2}, \frac{-1+y}{2}\right)$$

$$\Rightarrow -1 = \frac{3+x}{2} \quad \text{and} \quad 3 = \frac{-1+y}{2}$$

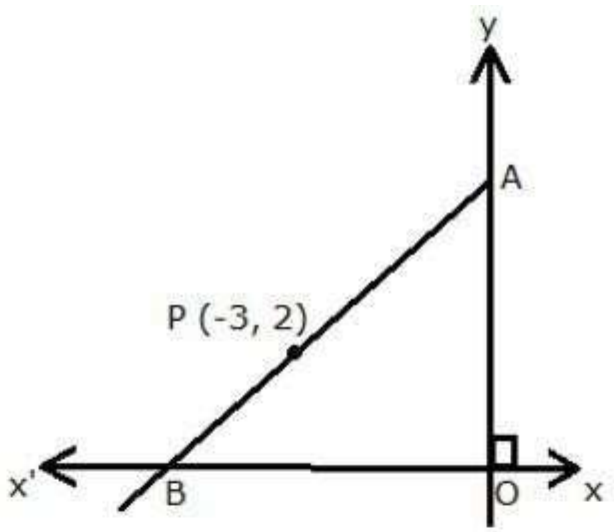
$$\Rightarrow -2 = 3+x \quad \text{and} \quad 6 = -1+y$$

$$\Rightarrow x = -5 \quad \text{and} \quad y = 7$$

Hence, the co-ordinates of B are (-5, 7).

Question 5.

P (-3, 2) is the mid-point of line segment AB as shown in the given figure. Find the co-ordinates of points A and B.



Solution:

Point A lies on y-axis, so let its co-ordinates be (0, y).
Point B lies on x-axis, so let its co-ordinates be (x, 0).
P (-3, 2) is the mid-point of line segment AB.

$$\therefore (-3,2) = \left(\frac{0+x}{2}, \frac{y+0}{2}\right)$$

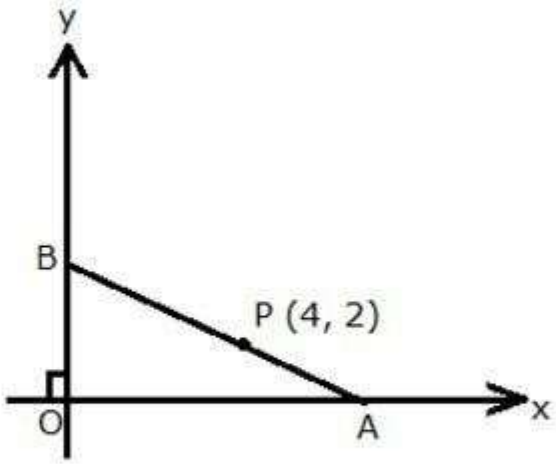
$$\Rightarrow (-3,2) = \left(\frac{x}{2}, \frac{y}{2}\right)$$

$$\Rightarrow -3 = \frac{x}{2} \quad \text{and} \quad 2 = \frac{y}{2}$$

$$\Rightarrow -6 = x \quad \text{and} \quad 4 = y$$

Thus, the co-ordinates of points A and B are (0, 4) and (-6, 0) respectively.

Question 6.
In the given figure, P (4, 2) is mid-point of line segment AB. Find the co-ordinates of A and B.



Solution:

Point A lies on x-axis, so let its co-ordinates be (x, 0).
Point B lies on y-axis, so let its co-ordinates be (0, y).
P (4, 2) is mid-point of line segment AB.

$$\therefore (4,2) = \left(\frac{x+0}{2}, \frac{0+y}{2}\right)$$

$$\Rightarrow 4 = \frac{x}{2} \quad \text{and} \quad 2 = \frac{y}{2}$$

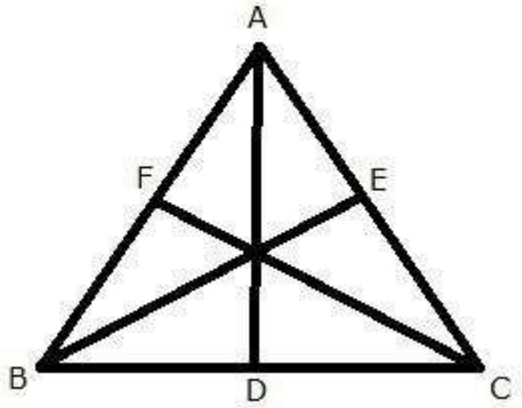
$$\Rightarrow 8 = x \quad \text{and} \quad 4 = y$$

Hence, the co-ordinates of points A and B are (8, 0) and (0, 4) respectively.

Question 7.
(-5, 2), (3, -6) and (7, 4) are the vertices of a triangle. Find the lengths of its median through the vertex (3, -6)

Solution:

Let A (-5, 2), B (3, -6) and C (7, 4) be the vertices of the given triangle.
Let AD be the median through A, BE be the median through B and CF be the median through C.



We know that median of a triangle bisects the opposite side.

Co-ordinates of point F are

$$\left(\frac{-5+3}{2}, \frac{2-6}{2}\right) = \left(\frac{-2}{2}, \frac{-4}{2}\right) = (-1, -2)$$

Co-ordinates of point D are

$$\left(\frac{3+7}{2}, \frac{-6+4}{2}\right) = \left(\frac{10}{2}, \frac{-2}{2}\right) = (5, -1)$$

Co-ordinates of point E are

$$\left(\frac{-5+7}{2}, \frac{2+4}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$

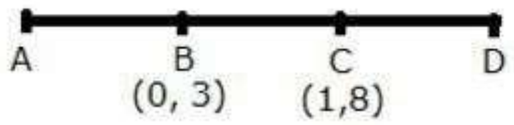
The median of the triangle through the vertex B(3, -6) is BE

Using distance formula,

$$BE = \sqrt{(1-3)^2 + (3+6)^2} = \sqrt{4+81} = \sqrt{85} = 9.22$$

Question 8.
Given a line ABCD in which AB = BC = CD, B = (0, 3) and C = (1, 8). Find the co-ordinates of A and D.

Solution:



Given, AB = BC = CD

So, B is the mid-point of AC. Let the co-ordinates of point A be (x, y).

$$\therefore (0, 3) = \left(\frac{x+1}{2}, \frac{y+8}{2}\right)$$

$$\Rightarrow 0 = \frac{x+1}{2} \quad \text{and} \quad 3 = \frac{y+8}{2}$$

$$\Rightarrow 0 = x+1 \quad \text{and} \quad 6 = y+8$$

$$\Rightarrow -1 = x \quad \text{and} \quad -2 = y$$

Thus, the co-ordinates of point A are (-1, -2).

Also, C is the mid-point of BD. Let the co-ordinates of point D be (p, q).

$$\therefore (1, 8) = \left(\frac{0+p}{2}, \frac{3+q}{2}\right)$$

$$\Rightarrow 1 = \frac{0+p}{2} \quad \text{and} \quad 8 = \frac{3+q}{2}$$

$$\Rightarrow 2 = 0+p \quad \text{and} \quad 16 = 3+q$$

$$\Rightarrow 2 = p \quad \text{and} \quad 13 = q$$

Thus, the co-ordinates of point D are (2, 13).

Question 9.
One end of the diameter of a circle is (-2, 5). Find the co-ordinates of the other end of it, if the centre of the circle is (2, -1).

Solution:

We know that the centre is the mid-point of diameter.
Let the required co-ordinates of the other end of mid-point be (x, y).

$$\therefore (2, -1) = \left(\frac{-2+x}{2}, \frac{5+y}{2}\right)$$

$$\Rightarrow 2 = \frac{-2+x}{2} \text{ and } -1 = \frac{5+y}{2}$$

$$\Rightarrow 4 = -2 + x \text{ and } -2 = 5 + y$$

$$\Rightarrow 6 = x \text{ and } -7 = y$$

Thus, the required co-ordinates are (6, -7).

Question 10.
A (2, 5), B (1, 0), C (-4, 3) and D (-3, 8) are the vertices of a quadrilateral ABCD. Find the co-ordinates of the mid-points of AC and BD.
Give a special name to the quadrilateral.

Solution:

Co-ordinates of the mid-point of AC are

$$\left(\frac{2-4}{2}, \frac{5+3}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$$

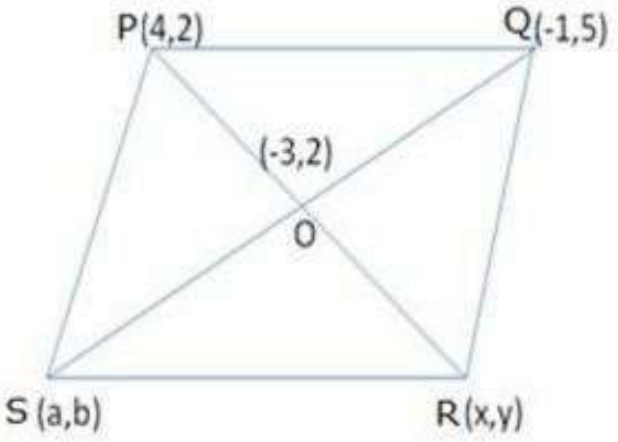
Co-ordinates of the mid-point of BD are

$$\left(\frac{1-3}{2}, \frac{0+8}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$$

Since, mid-point of AC = mid-point of BD
Hence, ABCD is a parallelogram.

Question 11.
P (4, 2) and Q (-1, 5) are the vertices of a parallelogram PQRS and (-3, 2) are the co-ordinates of the points of intersection of its diagonals. Find the coordinates of R and S.

Solution:



Let the coordinates of R and S be (x,y) and (a,b) respectively.
Mid-point of PR is O.

$\therefore O(-3,2) = O\left(\frac{4+x}{2}, \frac{2+y}{2}\right)$

$-3 = \frac{4+x}{2}, 2 = \frac{2+y}{2}$

$-6 = 4 + x, 4 = 2 + y$

$x = -10, y = 2$

Hence, R = (-10,2)

Similarly, the mid-point of SQ is O.

$\therefore O(-3,2) = O\left(\frac{a-1}{2}, \frac{b+5}{2}\right)$

$-3 = \frac{a-1}{2}, 2 = \frac{b+5}{2}$

$-6 = a - 1, 4 = b + 5$

$a = -5, b = -1$

Hence, $S = (-5, -1)$

Thus, the coordinates of the point R and S are (-10, 2) and (-5, -1).

Question 12.

A (-1, 0), B (1, 3) and D (3, 5) are the vertices of a parallelogram ABCD. Find the co-ordinates of vertex C.

Solution:

Let the co-ordinates of vertex C be (x, y).
ABCD is a parallelogram.
∴ Mid-point of AC = Mid-point of BD

$$\left(\frac{-1+x}{2}, \frac{0+y}{2}\right) = \left(\frac{1+3}{2}, \frac{3+5}{2}\right)$$

$$\left(\frac{-1+x}{2}, \frac{y}{2}\right) = (2, 4)$$

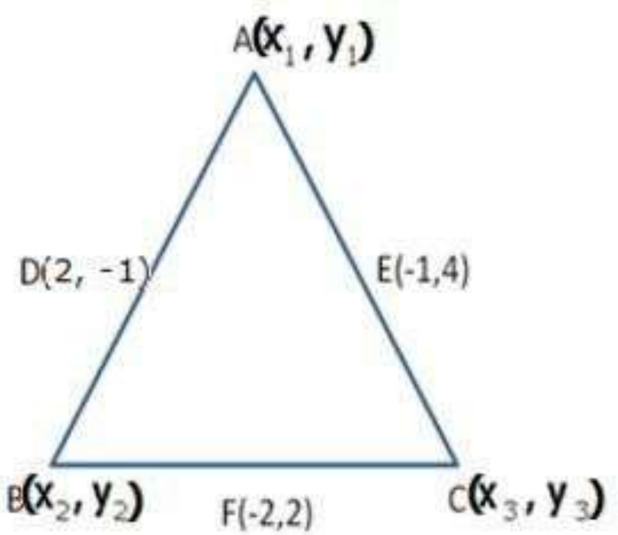
$$\frac{-1+x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = 4$$

$$x = 5 \quad \text{and} \quad y = 8$$

Thus, the co-ordinates of vertex C is (5, 8).

Question 13.
The points (2, -1), (-1, 4) and (-2, 2) are mid-points of the sides of a triangle. Find its vertices.

Solution:



Let $A(x_1,y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the co-ordinates of the vertices of ΔABC.

Midpoint of AB, i.e. D

$$D(2,-1) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$2 = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} = -1$$

$x_1 + x_2 = 4 - -(1)$ $y_1 + y_2 = -2 - -(2)$

Similarly,

$x_1 + x_3 = -2 - -(3)$ $y_1 + y_3 = 8 - -(4)$

$x_2 + x_3 = -4 - -(5)$ $y_2 + y_3 = 4 - -(6)$

Adding (1), (3) and (5), we get,

$2(x_1 + x_2 + x_3) = -2$

$x_1 + x_2 + x_3 = -1$

$4 + x_3 = -1[\text{from}(1)]$

$x_3 = -5$

From (3)

$x_1 - 5 = -2$

$x_1 = 3$

From (5)

$x_2 - 5 = -4$

$x_2 = 1$

Adding (2), (4) and (6), we get,

$2(y_1 + y_2 + y_3) = 10$

$y_1 + y_2 + y_3 = 5$

$-2 + y_3 = 5[\text{from}(2)]$

$y_3 = 7$

From (4)

$y_1 + 7 = 8$

$y_1 = 1$

From (6)

$y_2 + 7 = 4$

$y_2 = -3$

Thus, the co-ordinates of the vertices of ΔABC are (3, 1), (1, -3) and (-5, 7).

Question 14.
Points A (-5, x), B (y, 7) and C (1, -3) are collinear (i.e., lie on the same straight line) such that AB = BC. Calculates the values of x and y.

Solution:

Given, AB = BC, i.e., B is the mid-point of AC.

$$\therefore (y, 7) = \left(\frac{-5+1}{2}, \frac{x-3}{2}\right)$$

$$(y, 7) = \left(-2, \frac{x-3}{2}\right)$$

$$\Rightarrow y = -2 \quad \text{and} \quad 7 = \frac{x-3}{2}$$

$$\Rightarrow y = -2 \quad \text{and} \quad x = 17$$

Question 15.
Points P (a, -4), Q (-2, b) and R (0, 2) are collinear. If Q lies between P and R, such that PR = 2QR, calculate the values of a and b.

Solution:

Given, PR = 2QR
Now, Q lies between P and R, so, PR = PQ + QR
$\therefore$ PQ + QR = 2QR
$\Rightarrow$ PQ = QR
$\Rightarrow$ Q is the mid-point of PR.

$$\therefore (-2, b) = \left(\frac{a+0}{2}, \frac{-4+2}{2}\right)$$

$$(-2, b) = \left(\frac{a}{2}, -1\right)$$

$$\Rightarrow a = -4, \quad b = -1$$

Question 16.
Calculate the co-ordinates of the centroid of a triangle ABC, if A = (7, -2), B = (0, 1) and C = (-1, 4).

Solution:

Co-ordinates of the centroid of triangle ABC are

$$\left(\frac{7+0-1}{3}, \frac{-2+1+4}{3}\right)$$

$$= \left(\frac{6}{3}, \frac{3}{3}\right)$$

$$= (2, 1)$$

Question 17.
The co-ordinates of the centroid of a PQR are (2, -5). If Q = (-6, 5) and R = (11, 8); calculate the co-ordinates of vertex P.

Solution:
Let G be the centroid of DPQR whose coordinates are (2, -5) and let (x,y) be the coordinates of vertex P.

Coordinates of G are,

$$G(2, -5) = G\left(\frac{x-6+11}{3}, \frac{y+5+8}{3}\right)$$

$$2 = \frac{x+5}{3}, \quad -5 = \frac{y+13}{3}$$

$6 = x + 5, -15 = y + 13$

$x = 1, y = -28$

Coordinates of vertex P are (1, -28)

Question 18.
A (5, x), B (-4, 3) and C (y, -2) are the vertices of the triangle ABC whose centroid is the origin. Calculate the values of x and y.

Solution:
Given, centroid of triangle ABC is the origin.

$$\therefore (0, 0) = \left(\frac{5-4+y}{3}, \frac{x+3-2}{3}\right)$$

$$(0, 0) = \left(\frac{1+y}{3}, \frac{x+1}{3}\right)$$

$$0 = \frac{1+y}{3} \quad \text{and} \quad 0 = \frac{x+1}{3}$$

$$y = -1 \text{ and } \quad x = -1$$

Exercise 13C

Question 1.

Given a triangle ABC in which A = (4, -4), B = (0, 5) and C = (5, 10). A point P lies on BC such that BP: PC = 3: 2. Find the length of line segment AP.

Solution:

Given, BP: PC = 3: 2

Using section formula, the co-ordinates of point P are

$$\left(\frac{3\times 5+2\times 0}{3+2}, \frac{3\times 10+2\times 5}{3+2}\right)$$

$$=\left(\frac{15}{5}, \frac{40}{5}\right)$$

$$=(3,8)$$

Using distance formula, we have:

$$AP=\sqrt{(3-4)^2+(8+4)^2}=\sqrt{1+144}=\sqrt{145}=12.04$$

Question 2.

A (20, 0) and B (10, -20) are two fixed points. Find the co-ordinates of a point P in AB such that: 3PB = AB. Also, find the co-ordinates of some other point Q in AB such that AB = 6AQ.

Solution:

Given, 3PB = AB

$$\Rightarrow \frac{AB}{PB}=\frac{3}{1}$$

$$\Rightarrow \frac{AB-PB}{PB}=\frac{3-1}{1}$$

$$\Rightarrow \frac{AP}{PB}=\frac{2}{1}$$

Using section formula,

Coordinates of P are

$$P(x,y)=P\left(\frac{2\times 10+1\times 20}{2+1}, \frac{2\times(-20)+1\times 0}{2+1}\right)$$

$$= P\left(\frac{40}{3}, -\frac{40}{3}\right)$$

Given, AB = 6AQ

$$\Rightarrow \frac{AQ}{AB} = \frac{1}{6}$$

$$\Rightarrow \frac{AQ}{AB - AQ} = \frac{1}{6-1}$$

$$\Rightarrow \frac{AQ}{QB} = \frac{1}{5}$$

Using section formula,

Coordinates of Q are

$$Q(x, y) = Q\left(\frac{1 \times 10 + 5 \times 20}{1+5}, \frac{1 \times (-20) + 5 \times 0}{1+5}\right)$$

$$= Q\left(\frac{110}{6}, -\frac{20}{6}\right)$$

$$= Q\left(\frac{55}{3}, -\frac{10}{3}\right)$$

Question 3.

A (-8, 0), B (0, 16) and C (0, 0) are the vertices of a triangle ABC. Point P lies on AB and Q lies on AC such that AP: PB = 3: 5 and AQ: QC = 3: 5. Show that: PQ = 3/8 BC.

Solution:

Given that, point P lies on AB such that AP: PB = 3: 5.

The co-ordinates of point P are

$$\left(\frac{3 \times 0 + 5 \times (-8)}{3+5}, \frac{3 \times 16 + 5 \times 0}{3+5}\right)$$

$$= \left(\frac{-40}{8}, \frac{48}{8}\right)$$

$$= (-5, 6)$$

Also, given that, point Q lies on AB such that AQ: QC = 3: 5.

The co-ordinates of point Q are

$$\left(\frac{3 \times 0 + 5 \times (-8)}{3+5}, \frac{3 \times 0 + 5 \times 0}{3+5}\right)$$

$$= \left(\frac{-40}{8}, \frac{0}{8}\right)$$

$= (-5, 0)$

Using distance formula,

$PQ = \sqrt{(-5+5)^2 + (0-6)^2} = \sqrt{0+36} = 6$

$BC = \sqrt{(0-0)^2 + (0-16)^2} = \sqrt{0+(16)^2} = 16$

Now, $\frac{3}{8}BC = \frac{3}{8} \times 16 = 6 = PQ$

Hence, proved.

Question 4.
Find the co-ordinates of points of trisection of the line segment joining the point (6, -9) and the origin.

Solution:

Let P and Q be the points of trisection of the line segment joining A (6, -9) and B (0, 0).
P divides AB in the ratio 1: 2. Therefore, the co-ordinates of point P are

$\left(\frac{1 \times 0 + 2 \times 6}{1+2}, \frac{1 \times 0 + 2 \times (-9)}{1+2}\right)$

$= \left(\frac{12}{3}, \frac{-18}{3}\right)$

$= (4, -6)$

Q divides AB in the ratio 2: 1. Therefore, the co-ordinates of point Q are

$\left(\frac{2 \times 0 + 1 \times 6}{2+1}, \frac{2 \times 0 + 1 \times (-9)}{2+1}\right)$

$= \left(\frac{6}{3}, \frac{-9}{3}\right)$

$= (2, -3)$

Thus, the required points are (4, -6) and (2, -3).

Question 5.
A line segment joining A(-1, 5/3) and B (a, 5) is divided in the ratio 1: 3 at P, point where the line segment AB intersects the y-axis.
(i) Calculate the value of ‘a’.
(ii) Calculate the co-ordinates of ‘P’.

Solution:
Since, the line segment AB intersects the y-axis at point P, let the co-ordinates of point P

be (0, y).
P divides AB in the ratio 1: 3.

$$\therefore (0, y) = \left(\frac{1 \times a + 3 \times (-1)}{1+3}, \frac{1 \times 5 + 3 \times \frac{5}{3}}{1+3}\right)$$

$$(0, y) = \left(\frac{a-3}{4}, \frac{10}{4}\right)$$

$$0 = \frac{a-3}{4} \quad \text{and} \quad y = \frac{10}{4}$$

$$a = 3 \quad \text{and} \quad y = \frac{5}{2} = 2\frac{1}{2}$$

Thus, the value of a is 3 and the co-ordinates of point P are $\left(0, 2\frac{1}{2}\right)$.

Question 6.
In what ratio is the line joining A (0, 3) and B (4, -1) divided by the x-axis? Write the co-ordinates of the point where AB intersects the x-axis.

Solution:

Let the line segment AB intersects the x-axis by point P (x, 0) in the ratio k: 1.

$$\therefore (x, 0) = \left(\frac{k \times 4 + 1 \times 0}{k+1}, \frac{k \times (-1) + 1 \times 3}{k+1}\right)$$

$$(x, 0) = \left(\frac{4k}{k+1}, \frac{-k+3}{k+1}\right)$$

$$\Rightarrow 0 = \frac{-k+3}{k+1}$$

$$\Rightarrow k = 3$$

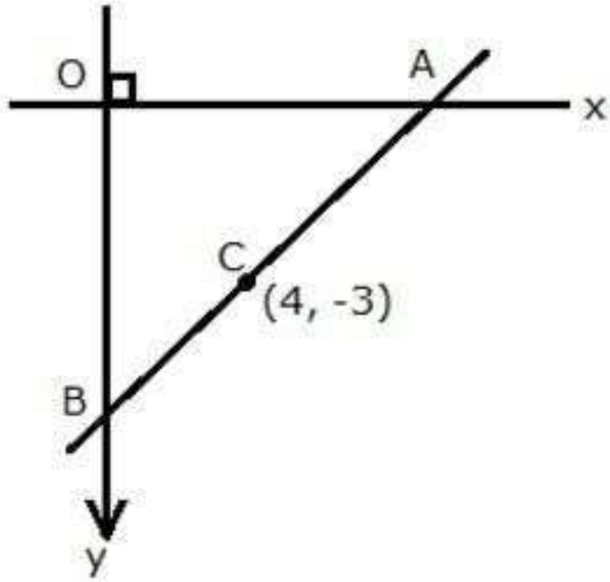
Thus, the required ratio in which P divides AB is 3: 1.
Also, we have:

$$x = \frac{4k}{k+1}$$

$$\Rightarrow x = \frac{4 \times 3}{3+1} = \frac{12}{4} = 3$$

Thus, the co-ordinates of point P are (3, 0).

Question 7.
The mid-point of the segment AB, as shown in diagram, is C (4, -3). Write down the co-ordinates of A and B.



Solution:

Since, point A lies on x-axis, let the co-ordinates of point A be (x, 0).
Since, point B lies on y-axis, let the co-ordinates of point B be (0, y).
Given, mid-point of AB is C (4, -3).

$$\therefore (4,-3) = \left(\frac{x+0}{2}, \frac{0+y}{2}\right)$$

$$\Rightarrow (4,-3) = \left(\frac{x}{2}, \frac{y}{2}\right)$$

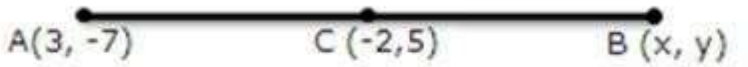
$$\Rightarrow 4 = \frac{x}{2} \quad \text{and} \quad -3 = \frac{y}{2}$$

$$\Rightarrow x = 8 \quad \text{and} \quad y = -6$$

Thus, the co-ordinates of point A are (8, 0) and the co-ordinates of point B are (0, -6).

Question 8.
AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7), find
(i) the length of radius AC
(ii) the coordinates of B.

Solution:



(i) Radius $AC = \sqrt{(3+2)^2 + (-7-5)^2}$

$$= \sqrt{5^2 + (-12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

(ii) Let the coordinates of B be (x, y).

Using mid – point formula, we have

$$-2 = \frac{3+x}{2} \quad \text{and} \quad 5 = \frac{-7+y}{2}$$
$$\Rightarrow -4 = 3 + x \quad \text{and} \quad 10 = -7 + y$$
$$\Rightarrow x = -7 \quad \text{and} \quad y = 17$$

Thus, the coordinates of B are (−7, 17).

Question 9.
Find the co-ordinates of the centroid of a triangle ABC whose vertices are:
A (-1, 3), B (1, -1) and C (5, 1)

Solution:

Co- ordinates of the centroid of triangle ABC are

$$\left(\frac{-1+1+5}{3}, \frac{3-1+1}{3}\right)$$
$$= \left(\frac{5}{3}, 1\right)$$

Question 10.
The mid-point of the line-segment joining (4a, 2b – 3) and (-4, 3b) is (2, -2a). Find the values of a and b.

Solution:
It is given that the mid-point of the line-segment joining (4a, 2b – 3) and (-4, 3b) is (2, -

2a).

$$\therefore (2, -2a) = \left(\frac{4a-4}{2}, \frac{2b-3+3b}{2}\right)$$

$$\Rightarrow 2 = \frac{4a-4}{2}$$

$$\Rightarrow 4a - 4 = 4$$

$$\Rightarrow 4a = 8$$

$$\Rightarrow a = 2$$

Also,

$$-2a = \frac{2b-3+3b}{2}$$

$$\Rightarrow -2 \times 2 = \frac{5b-3}{2}$$

$$\Rightarrow 5b - 3 = -8$$

$$\Rightarrow 5b = -5$$

$$\Rightarrow b = -1$$

Question 11.
The mid-point of the line segment joining (2a, 4) and (-2, 2b) is (1, 2a + 1). Find the value of a and b.

Solution:

Mid-point of (2a, 4) and (-2, 2b) is (1, 2a + 1), therefore using mid-point formula, we have:

$$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2}$$

$$1 = \frac{2a-2}{2} \quad 2a + 1 = \frac{4 + 2b}{2}$$

$$1 = a - 1$$

$$\therefore a = 2 \quad 2a + 1 = 2 + b$$

Putting, a = 2 in 2a + 1 = 2 + b, we get,
5 - 2 = b ⇒ b = 3
Therefore, a = 2, b = 3.

Question 12.
(i) Write down the co-ordinates of the point P that divides the line joining A (-4, 1) and B (17, 10) in the ratio 1: 2.
(ii) Calculate the distance OP, where O is the origin.
(iii) In what ratio does the y-axis divide the line AB?

Solution:

(i) Co-ordinates of point P are

$$\left(\frac{1\times 17+2\times(-4)}{1+2},\frac{1\times 10+2\times 1}{1+2}\right)$$
$$=\left(\frac{17-8}{3},\frac{10+2}{3}\right)$$
$$=\left(\frac{9}{3},\frac{12}{3}\right)$$
$$=(3,4)$$

(ii) $OP=\sqrt{(0-3)^2+(0-4)^2}=\sqrt{9+16}=\sqrt{25}=5$ units

(iii) Let AB be divided by the point P (0, y) lying on y-axis in the ratio k: 1.

$$\therefore (0,y)=\left(\frac{k\times 17+1\times(-4)}{k+1},\frac{k\times 10+1\times 1}{k+1}\right)$$
$$\Rightarrow (0,y)=\left(\frac{17k-4}{k+1},\frac{10k+1}{k+1}\right)$$
$$\Rightarrow 0=\frac{17k-4}{k+1}$$
$$\Rightarrow 17k-4=0$$
$$\Rightarrow k=\frac{4}{17}$$

Thus, the ratio in which the y-axis divide the line AB is 4: 17.

Question 13.
Prove that the points A (-5, 4), B (-1, -2) and C (5, 2) are the vertices of an isosceles right-angled triangle. Find the co-ordinates of D so that ABCD is a square.

Solution:

We have:

$$AB = \sqrt{(-1+5)^2 + (-2-4)^2} = \sqrt{16+36} = \sqrt{52}$$
$$BC = \sqrt{(-1-5)^2 + (-2-2)^2} = \sqrt{36+16} = \sqrt{52}$$
$$AC = \sqrt{(5+5)^2 + (2-4)^2} = \sqrt{100+4} = \sqrt{104}$$
$$AB^2 + BC^2 = 52 + 52 = 104$$
$$AC^2 = 104$$

$\because$ AB = BC and $AB^2 + BC^2 = AC^2$
$\therefore$ ABC is an isosceles right-angled triangle.

Let the coordinates of D be (x, y).
If ABCD is a square, then,
Mid-point of AC = Mid-point of BD

$$\left(\frac{-5+5}{2}, \frac{4+2}{2}\right) = \left(\frac{x-1}{2}, \frac{y-2}{2}\right)$$
$$0 = \frac{x-1}{2}, 3 = \frac{y-2}{2}$$

x = 1, y = 8
Thus, the co-ordinates of point D are (1, 8).

Question 14.
M is the mid-point of the line segment joining the points A (-3, 7) and B (9, -1). Find the co-ordinates of point M. Further, if R (2, 2) divides the line segment joining M and the origin in the ratio p: q, find the ratio p: q.

Solution:

Given, M is the mid-point of the line segment joining the points A (-3, 7) and B (9, -1).
The co-ordinates of point M are

$$\left(\frac{-3+9}{2}, \frac{7-1}{2}\right)$$
$$= \left(\frac{6}{2}, \frac{6}{2}\right)$$
$$= (3, 3)$$

Also, given that, R (2, 2) divides the line segment joining M and the origin in the ratio p: q.

$$\therefore (2,2) = \left(\frac{p\times 0 + q\times 3}{p+q}, \frac{p\times 0 + q\times 3}{p+q}\right)$$

$$\Rightarrow \frac{p\times 0 + q\times 3}{p+q} = 2$$

$$\Rightarrow \frac{3q}{p+q} = 2$$

$$\Rightarrow 3q = 2p + 2q$$

$$\Rightarrow 3q - 2q = 2p$$

$$\Rightarrow q = 2p$$

$$\Rightarrow \frac{p}{q} = \frac{1}{2}$$

Thus, the ratio p: q is 1: 2.

Question 15.

Calculate the ratio in which the line joining A(-4, 2) and B(3, 6) is divided by point P(x, 3). Also, find

(a) x

(b) length of AP.

Solution:

Let P(x, 3) divides the line segment joining the points A(−4, 2) and B(3, 6) in the ratio k : 1.

Thus, we have

$$\frac{3k-4}{k+1} = x; \qquad \frac{6k+2}{k+1} = 3$$

For

$$6k + 2 = 3(k+1)$$

$$\Rightarrow 6k + 2 = 3k + 3$$

$$\Rightarrow 3k = 3 - 2$$

$$\Rightarrow 3k = 1 \Rightarrow k = \frac{1}{3}$$

$\therefore$ Required ratio is 1 : 3.

a. Now consider the equation $\frac{3k-4}{k+1} = x$

Substituting the value of k in the above equation, we have

$$\frac{3\times\frac{1}{3}-4}{\frac{1}{3}+1} = x \Rightarrow \frac{-3}{\frac{4}{3}} = x \Rightarrow \frac{-9}{4} = x$$

$$\therefore x = -\frac{9}{4}$$

b. $AP = \sqrt{\left(\frac{-9}{4}+4\right)^2 + (3-2)^2} = \sqrt{\frac{49}{16}+1} = \sqrt{\frac{49+16}{16}} = \sqrt{\frac{65}{16}}$

$$\Rightarrow AP = \frac{\sqrt{65}}{4} \text{ units}$$

Question 16.

Find the ratio in which the line 2x + y = 4 divides the line segment joining the points P(2, -2) and Q(3, 7).

Solution:

Let the line $2x + y = 4$ divides the line segment joining the points P(2, −2) and Q(3, 7) in the ratio k : 1.

Then, we have

$$(x, y) = \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

Since $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$ lies on line $2x + y = 4$, we have

$$2\left(\frac{3k+2}{k+1}\right) + \frac{7k-2}{k+1} = 4$$

$$\Rightarrow 6k + 4 + 7k - 2 = 4k + 4$$

$$\Rightarrow 13k + 2 = 4k + 4$$

$$\Rightarrow 9k = 2$$

$$\Rightarrow k = \frac{2}{9}$$

Hence, required ratio is 2 : 9.

Question 17.
If the abscissa of a point P is 2. Find the ratio in which this point divides the line segment joining the point (-4, 3) and (6, 3). Al so, find the co-ordinates of point P.

Solution:

Let the point P divides the line segment joining the points A(−4, 3) and B(6, 3) in the ratio k : 1.
Then, we have

$$(2, y) = \left(\frac{6k-4}{k+1}, \frac{3k+3}{k+1}\right)$$

$$\Rightarrow \frac{6k-4}{k+1} = 2$$

$$\Rightarrow 6k - 4 = 2k + 2$$

$$\Rightarrow 4k = 6$$

$$\Rightarrow k = \frac{3}{2}$$

∴ Required ratio is 3 : 2.

Also, $\frac{3k+3}{k+1} = y$

$$\Rightarrow \frac{3 \times \frac{3}{2} + 3}{\frac{3}{2} + 1} = y$$

$$\Rightarrow \frac{15/2}{5/2} = y$$

$$\Rightarrow y = 3$$

Hence, coordinates of point P are (2, 3).

Question 18.
The line joining the points (2, 1) and (5, -8) is trisected at the points P and Q, point P lies on the line 2x – y + k = 0, find the value of k. Also, find the co-ordinates of point Q.

Solution:

Let A(2, 1) and B(5, − 8) be the given points trisected by the points P and Q.

$\Rightarrow AP = PQ = QB$

For P :

$m_1 : m_2 = AP : PB = 1 : 2$

$(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (5, -8)$

$\therefore\ x = \dfrac{1 \times 5 + 2 \times 2}{1 + 2} = \dfrac{5 + 4}{3} = \dfrac{9}{3} = 3$

$y = \dfrac{1 \times (-8) + 2 \times 1}{1 + 2} = \dfrac{-8 + 2}{3} = \dfrac{-6}{3} = -2$

$\therefore$ Coordinates of P are (3, − 2).

Since point P lies on the line $2x - y + k = 0$, we have

$2(3) - (-2) + k = 0$

$\Rightarrow 6 + 2 + k = 0$

$\Rightarrow k = -8$

For Q:

$m_1 : m_2 = AQ : QB = 2 : 1$

$(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (5, -8)$

$\therefore\ x = \dfrac{2 \times 5 + 1 \times 2}{2 + 1} = \dfrac{10 + 2}{3} = \dfrac{12}{3} = 4$

$y = \dfrac{2 \times (-8) + 1 \times 1}{1 + 2} = \dfrac{-16 + 1}{3} = \dfrac{-15}{3} = -5$

$\therefore$ Coordinates of Q are (4, − 5).

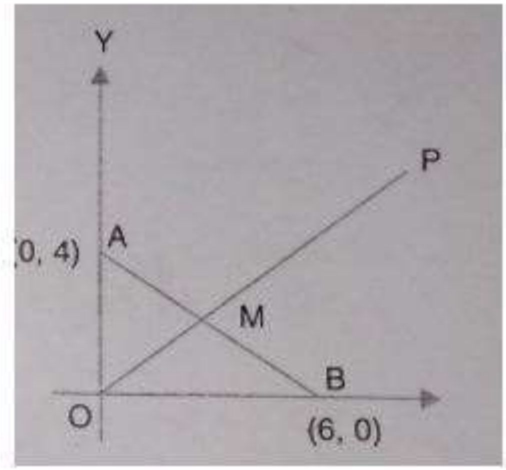
Question 19.

M is the mid-point of the line segment joining the points A(0, 4) and B(6, 0). M also divides the line segment OP in the ratio 1 : 3. Find :

(a) co-ordinates of M

(b) co-ordinates of P

(c) length of BP



Solution:

a. M is the mid-point of line segment joining points A(0, 4) and B(6, 0).

$\therefore M = \left(\frac{0+6}{2}, \frac{4+0}{2}\right) = (3, 2)$

b. M divides OP in the ratio 1 : 3

Let the coordinates of P be (x, y).

$\therefore M = \left(\frac{x+0}{1+3}, \frac{y+0}{1+3}\right) = \left(\frac{x}{4}, \frac{y}{4}\right)$

But, M = (3, 2)

$\therefore \frac{x}{4} = 3$ and $\frac{y}{4} = 2$

$\Rightarrow x = 12$ and $y = 8$

$\therefore$ Coordinates of P are (12, 8).

c. $BP = \sqrt{(12-6)^2 + (8-0)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ units

Question 20.
Find the image of the point A(5, -3) under reflection in the point P(-1, 3).

Solution:

Let the image be B(x,y).

Since A is reflected in P, P is the mid-point of AB.

Using Mid-point Formula, we get

$$\Rightarrow P(-1,3) = \left(\frac{x+5}{2}, \frac{y-3}{2}\right)$$

$$\Rightarrow \frac{x+5}{2} = -1 \text{ and } \frac{y-3}{2} = 3$$

$$\Rightarrow x+5 = -2 \text{ and } y-3 = 6$$

$$\Rightarrow x = -7 \text{ and } y = 9$$

So, the image of A in P is B(-7,9).

Question 21.

A(-4, 2), B(0, 2) and C(-2, -4) are the vertices of a triangle ABC. P, Q and R are mid-points of sides BC, CA and AB respectively. Show that the centroid of Δ PQR is the same as the centroid of Δ ABC.

Solution:

A(-4, 2), B(0, 2) and C(-2, -4) are the vertices of ΔABC.

$$\therefore \text{Centroid of } \Delta ABC = \left(\frac{-4+0-2}{3}, \frac{2+2-4}{3}\right) = \left(\frac{-6}{3}, \frac{0}{3}\right) = (-2, 0)$$

P, Q and R are the mid-points of sides BC, CA and AB respectively.

$$\therefore \text{Coordinates of } P = \left(\frac{0-2}{2}, \frac{2-4}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1)$$

$$\text{Coordinates of } Q = \left(\frac{-2-4}{2}, \frac{-4+2}{2}\right) = \left(\frac{-6}{2}, \frac{-2}{2}\right) = (-3, -1)$$

$$\text{Coordinates of } R = \left(\frac{-4+0}{2}, \frac{2+2}{2}\right) = \left(\frac{-4}{2}, \frac{4}{2}\right) = (-2, 2)$$

$$\therefore \text{Centroid of } \Delta PQR = \left(\frac{-1-3-2}{3}, \frac{-1-1+2}{3}\right) = \left(\frac{-6}{3}, \frac{0}{3}\right) = (-2, 0)$$

$$\Rightarrow \text{Centroid of } \Delta ABC = \text{Centroid of } \Delta PQR.$$

Question 22.

Centroid of $\Delta ABC = \left(\frac{3+y+1}{3}, \frac{1+4+x}{3}\right) = \left(\frac{4+y}{3}, \frac{5+x}{3}\right)$ ... (i)

P, Q and R are the mid points of the sides BC, CA and AB.

By mid - point formula, we get

$\Rightarrow P = \left(\frac{y+1}{2}, \frac{4+x}{2}\right),\ Q = \left(\frac{4}{2}, \frac{1+x}{2}\right)$ and $R = \left(\frac{3+y}{2}, \frac{5}{2}\right)$

$$\therefore \text{ Centroid of a } \Delta PQR = \left(\frac{\frac{y+1}{2}+\frac{4}{2}+\frac{3+y}{2}}{3}, \frac{\frac{4+x}{2}+\frac{1+x}{2}+\frac{5}{2}}{3}\right)$$

$$= \left(\frac{\frac{y+1+4+3+y}{2}}{3}, \frac{\frac{4+x+1+x+5}{2}}{3}\right)$$

$$= \left(\frac{8+2y}{6}, \frac{10+2x}{6}\right)$$

$$= \left(\frac{4+y}{3}, \frac{5+x}{3}\right) \quad \text{...... (ii)}$$

From (i) and (ii), we get

Centroid of a ΔABC = Centroid of a ΔPQR